ATUL CLASSES

Test / Exam Name: Mensuration | **Standard: 10th** | **Subject: Mathematics**

Student Name: ____________ **Section:** ________ **Roll No.:** ____________

Questions: 15 | **Time: 01:00 hh:mm** | **Marks: 75**

Q1. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36cm, 25cm and 16.5cm provided the thickness of the iron is 1.5cm. If one cubic cm of iron weighs 7.5g, find the weight of the box. **5 Marks**

Ans:

External dimenesions

l_2 = 39cm

b_2 = 25cm

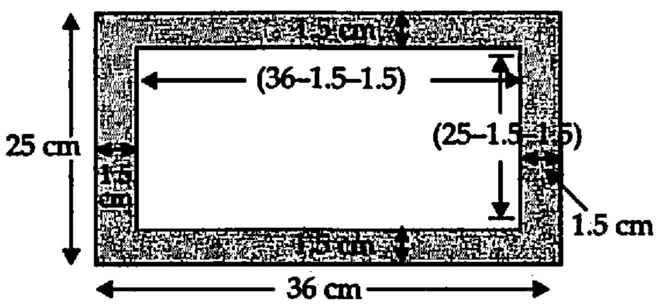
h_2 = 16.5cm

Internal dimenesions

l_1 = 36 - 1.5 - 1.5 = 36 - 3 = 33cm

b_1 = 25 - 3 = 22cm

h_1 = 16.5 - 15 = 15cm



Volume of iron in the open box

= $l_2b_2h_2$ - $l_1b_1h_1$

= (36 × 25 × 16.5) - (33 × 22 × 15)

⇒ Volume of iron in the open box

$= 9 \times 5 \times 11\left[\frac{4 \times 5 \times 15}{10} - 22\right]$

$= 45 \times 11[30 - 22] = 495 \times 8 = 3960\text{cm}^3$

Volume of iron is 3960cm^3.

So, 3960cm^3 of iron will weigh $\frac{3960 \times 75}{10} = 396 \times 75\text{gm}$

$= \frac{396 \times 75}{1000}\text{kg} = \frac{297}{10}\text{kg}$

Hence, the weight of the box = 29.7kg.

Q2. Marbles of diameter 1.4cm are dropped into a cylindrical beaker of diameter 7cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6cm. **5 Marks**

Ans:

Given, diameter of a marble = 1.4cm

∴ Radius of marble $= \frac{1.4}{2} = 0.7\text{cm}$

So, volume of one marble $= \frac{4}{3}\pi(0.7)^3$

$= \frac{4}{3}\pi \times 0.343 = \frac{1.372}{3}\pi\ \text{cm}^3$

Also, given diameter of beaker = 7cm

∴ Radius of beaker $= \frac{7}{2} = 3.5\text{cm}$

Height of water level raised = 5.6cm

∴ Volume of the raised water in becaker $= \pi(3.5)^2 \times 5.6 = 68.6\pi\ \text{cm}^3$

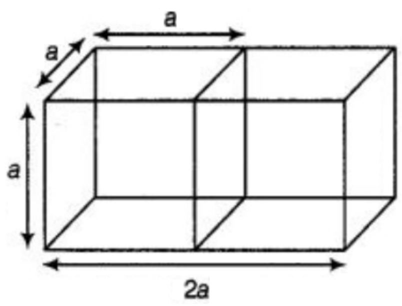
Now, required number of marbles $= \frac{\text{Volume of the raised water in beaker}}{\text{Volume of one spherical marble}}$

$= \frac{68.6\pi}{1.372\pi} \times 3 = 150$

Q3. Two identical cubes each of volume 64cm^3 are joined together end to end. What is the surface area of the resulting cuboid? **5 Marks**

Ans:

Let the length of side of a cube = a cm



Given, volume of the cube, $a^3 = 64cm^3$

$\Rightarrow$ a = 4cm

On joining two cubes, we get a cuboid whose

length, l = 2a cm

breadth, b = a cm

and height, h = a cm

Now, surface area of the resulting cuboid = 2(lb + bh + hl)

= 2(2a.a + a.a + a.2a)

$= 2(2a^2 + a^2 + 2a^2) = 2(5a^2)$

$= 10a^2 = 10(4)^2 = 160cm^2$

Q4. A solid rectangular block of dimensions 4.4m, 2.6m and 1m is cast into a hollow cylindrical pipe of internal radius 30cm and thickness 5cm. Find the length of the pipe. **5 Marks**

Ans:

We have,

Length of the rectangular block, l = 4. 4m,

Breadth of the rectangular block, b = 2. 6m,

Height of the rectangular block, h = 1m,

Internal radius of the cylindrical pipe, r = 30cm = 0. 3m and

Thickness of the pipe = 5cm = 0. 05m

Also, the external radius of the pipe = 0. 3 + 0. 05 = 0. 35m

Let the length of the pipe be H.

Now,

Volume of the pipe = Volume of the block

$\Rightarrow \pi R^2 H = \pi r^2 H = lbh$

$\Rightarrow \pi(R^2 - r^2)H = lbh$

$\Rightarrow \frac{22}{7} \times (0.35^2 - 0.3^2)H = 4.4 \times 2.6 \times 1$

$\Rightarrow \frac{22}{7} \times (0.1225 - 0.09)H = 4.4 \times 2.6$

$\Rightarrow \frac{22}{7} \times 0.0325 \times H = 4.4 \times 2.6$

$= H = \frac{4.4 \times 2.6 \times 7}{22 \times 0.0325}$

$\therefore H = 112m$

So, the length of the pipe is 112m.

Q5. A tent of height 77dm is in the form a right circular cylinder of diameter 36m and height 44dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 3.50 per m^2. $\left[\text{use}\pi = \frac{22}{7}\right]$ **5 Marks**

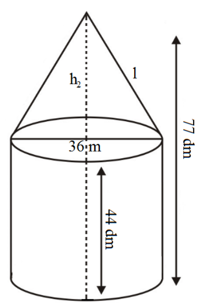
Ans:

Total height of the tent = 77dm

Height of the cylindrical part (h_1) = 44dm = 4.4m

Height of conical part (h_2) = 77 – 44 = 33dm = 3.3m

Diameter of the base of the tent = 36m



$\therefore$ Radius (r) $= \frac{36}{2} = 18m$

$\therefore$ Slant height of the conical part $= \sqrt{r^2 + {h_2}^2}$

$l = \sqrt{18^2 + (3.3)^2} = \sqrt{324 + 11} = \sqrt{335} = 18.30m$

C.S.A of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 18 \times 4.4$

$= \frac{3484.8}{7}$

$= 497.82$

$= 498m^2$

S.A of cone $= \pi rl$

$= \frac{22}{7} \times 18 \times 18.30$

$= 1035.25$

Total area = Area of cylinder + Area of cone

$= 498 + 1035.25$

$= 1533.25m^2$

Cost of canvas $= 1533.25 \times 3.50$

$= 5366.4$

Q6. Three cubes of a metal whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$cm Find the edges of the three cubes. **5 Marks**

Ans:

Let the edge of the metal cubes be 3x, 4x and 5x.

Let the edge of the single cube be a.

As,

Diagonal of the single cube $12\sqrt{3}$cm

$\Rightarrow a\sqrt{3} = 12\sqrt{3}$

$\Rightarrow a = 12cm$

Now,

Volume of the single cube = Sum of the volumes of the metallic cubes

$\Rightarrow a^3 = (3x)^3 + (4x)^3 + (5x)^3$

$\Rightarrow 12^3 = 27x^3 + 64x^3 + 125x^3$

$\Rightarrow 1728 = 216x^3$

$\Rightarrow x^3 = \frac{1728}{216}$

$\Rightarrow x^3 = 8$

$\Rightarrow x = \sqrt[3]{8}$

$\Rightarrow x = 2$

So, the egdes of the cubes are 3 × 2 = 6cm, 4 × 2 = 8cm and 5 × 2 = 10cm.

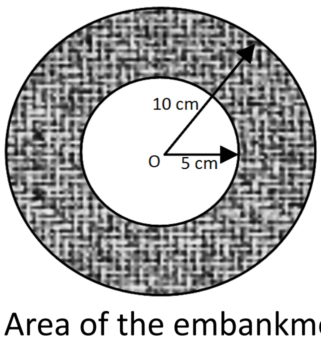
Hence, the edges of the given three metallic cubes are 6cm, 8cm and 10cm.

Q7. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment. **5 Marks**

Ans:

Let r m be the radius and h m be the height of the well (cylindrical shape). Then,

r = 1.5m and h = 14m



Area of the embankment (shnded part)

$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

$= \frac{22}{7}[(5.5)^2 - (1.5)^2]$

$= \frac{22}{7}(5.5 + 1.5)(5.5 - 1.5)$

$= \left(\frac{22}{7} \times 7 \times 4\right)\text{cm}^2$

$= 88\text{cm}^2$

$\therefore$ Height of the embankment

$= \frac{\text{Volume of the earth dug out}}{\text{Area of the embankment}}$

$= \frac{99}{88} = 1.125\text{m}$

Q8. How many spherical lead shots each of diameter 4.2cm can be obtained from a solid rectangular lead piece with dimensions 66cm, 42cm and 21cm. **5 Marks**

Ans:

Given that, lots of spherical lead shots made from a solid rectangular lead piece.

$\therefore$ Number of spherical lead shots $= \frac{\text{Volume of solid rectangular lead piece}}{\text{Volume of a spherical lead shot}} \ldots\text{(i)}$

Also, given that diameter of a spherical lead shot i.e., sphere = 4.2cm

$\therefore$ Radius of a spherical lead shot, $\text{r} = \frac{42}{2} = 2.1\text{cm} \left[\because \text{radius} = \frac{1}{2}\text{diameter}\right]$

So, volume of a spherical lead shot i.e., sphere

$= \frac{4}{3}\pi\text{r}^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$

$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 21$

$= \frac{4 \times 22 \times 21 \times 21 \times 21}{3 \times 7 \times 1000}$

Now, length of rectangular lead piece, l = 66cm

Breadth of rectangular lead piece, b = 42cm

Height of rectagular lead piece, h = 21cm

$\therefore$ Volume of a solid rectangular lead piece i.e., cuboid = l × b × h = 66 × 42 × 21

From Eq. (i),

Number of spherical lead shots $= \frac{66 \times 42 \times 21}{4 \times 22 \times 21 \times 21 \times 21} \times 3 \times 7 \times 1000$

$= \frac{3 \times 22 \times 21 \times 2 \times 21 \times 21 \times 1000}{4 \times 22 \times 21 \times 21 \times 21}$

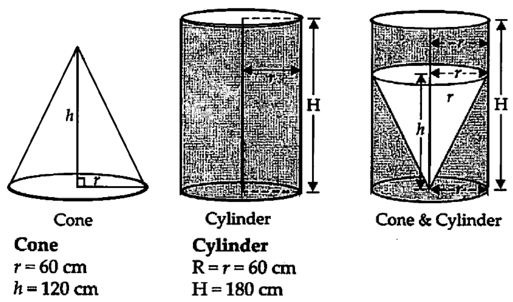
$= 3 \times 2 \times 250$

$= 6 \times 250 = 1500$

Hence, the required number of special lead shots is 1500.

Q9. A solid right circular cone of height 120cm and radius 60cm is placed in a right circular cylinder full of water of height 180cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone. **5 Marks**

Ans:



Cone is placed inside the cylindrical vessel full of water.

So, the volume of water from cylinder will over flow equal to the volume of cone.

Hence, the water left in cylinder = Volume of cylinder - Volume of cone (i)

Volume of water left after immersing the cone into cylinder full of water = Volume of cylider - Volume of cone

$= \pi\text{R}^2\text{H} - \frac{1}{3}\pi\text{r}^2\text{h}$

$\therefore$ Required volume of water in cylinder

$= \pi\text{r}^2\text{H} - \frac{1}{3}\pi\text{r}^2\text{h} \; [\because \text{R} = \text{r}]$

$= \pi\text{r}^2\left[\text{H} - \frac{1}{3}\text{h}\right] = \frac{22}{7} \times 60 \times 60\left[180 - \frac{120}{3}\right]$

$= \frac{22}{7} \times 60 \times 60 \times 140\text{cm}^3$

$= \frac{22 \times 60 \times 60 \times 140}{7 \times 100 \times 100 \times 100} = \frac{22 \times 72}{1000} = \frac{1584}{1000}$

$\therefore$ Volume of water in cylinder = 1.584m^3

Hence, required volume of water left = 1.584m^3.

Q10. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream. **5 Marks**

Ans:

Let R cm be the radius and H cm be the height of a container, then

$\mathrm{R} = \frac{12}{2} = 6\mathrm{cm}$ and H = 15 cm

Therefore, volume of cylindrical container

$= \pi r^2 H$

$= (\pi \times 6 \times 6 \times 15)\mathrm{cm}^3$

$= 540\pi\ \mathrm{cm}^3$

Let r_1 cm be the radius and h cm be the height of a cone, then,

$r_1 = \frac{6}{2} = 3\mathrm{cm}$ and h = 12 cm

Therefore, Volume of conical part

$= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3}\pi \times 3 \times 3 \times 12\right)\mathrm{cm}^3$

$= 36\pi\ \mathrm{cm}^3$

Let r_2 cm be the radius of hemispherical part then r_2 = 3 cm, [$\because r_1 = r_2$]

Therefore, Volume of Hemispherical part

$= \frac{2}{3}\pi r_1^2 h$

$= \left(\frac{2}{3} \times \pi \times 3 \times 3 \times 3\right)\mathrm{cm}^3$

$= 18\pi\ \mathrm{cm}^3$

Now, Volume of ice-creme cone with hemispherical top

= Volume of cone + Volume of Hemisphere

$= (36\pi + 18\pi)\mathrm{cm}^3$

$= 54\pi\ \mathrm{cm}^3$

Therefore,

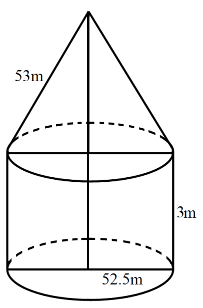
The required no. of such cones

$= \frac{\text{Volume of cylindrical container}}{\text{Vo I um e of cone with hemispherical top}}$

$= \frac{540\pi}{54\pi} = 10$

Q11. A circus tent is cylindrical to a height of 3m and conical above it. If its base radius is 52.5m and the slant height of the conical portion is 53m find the area of canvas needed to make the tent. $\left[\text{Take } \pi = \frac{22}{7}\right]$ **5 Marks**

Ans:



For the cylindrical portion, we have radius = 52.2m and height = 3m

For the conocal poetion, we have radius = 52.5m

And slant height = 53m

Area of canvas $= 2\pi rh + \pi l(2h + l)$

$= \left[\frac{22}{7} \times 52.5 \times (2 \times 3 + 53)\right]\mathrm{m}^2$

$= \left(22 \times \frac{15}{2} \times 59\right)\mathrm{m}^2 = 9735\mathrm{m}^2$

Q12. A spherical ball of radius 3cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5cm and 2cm. Find the diameter of the third ball. **5 Marks**

Ans:

Radius of big spherical ball (R) = 3cm

$\therefore$ volume $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (3)^3 cm^3$

$= \frac{4}{3}\pi \times 27 = 36\pi cm^3$

Similarly volume of ball of radius (r_1) = 1.5cm

$\therefore$ volume $= \frac{4}{3}\pi(1.5)^3$

$= \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 cm^3$

$= \frac{4}{3}\pi \times \frac{27}{8} = \frac{9\pi}{2} cm^3$

And Volume of ball of radius (r_2) = 2cm

$= \frac{4}{3}\pi \times \pi(2)^3$

$= \frac{4}{3}\pi \times 8 = \frac{32}{3}\pi cm^3$

$\therefore$ Volume of third ball $= 36\pi - \left(\frac{9}{2}\pi + \frac{32}{3}\pi\right)$

$= 36\pi - \left(\frac{27+64}{6}\pi\right)$

$= 36\pi - \frac{91}{6}\pi$

$= \frac{216\pi - 91\pi}{6} = \frac{125}{6}\pi cm^3$

$\therefore$ radius of the third ball

$= 3\sqrt{\frac{125}{6}\pi \times \frac{3}{4\pi}} = 3\sqrt{\frac{125}{8}}$

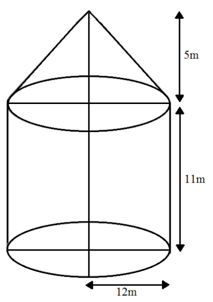
$= 3\sqrt{\left(\frac{5}{2}\right)^3} = \frac{5}{2} cm = 2.5cm$

$\therefore$ diameter $= 2 \times$ radius

$= 2 \times 2.5 = 5cm$

Q13. A cylindrical road roller made of iron is 1m long, Its internal diameter is 54cm and the thickness of the iron sheet used in making the roller is 9cm. Find the mass of the roller, if 1cm^3 of iron has 7.8gm mass. (use $\pi = 3.14$) **5 Marks**

Ans:



Given internal radius of cylinder road roller (r_1) $= \frac{54}{2} = 27cm$

Given thickness of road roller $\left(\frac{1}{b}\right) = 9cm$

Let order radii of cylinderical road roller be R.

⇒ t = R - r

⇒ 9 = R - 27

⇒ R = 9 + 27 = 36cm

⇒ R = 36cm

Given height of cylindrical road roller (h) = 1m

h = 100cm

Volume of iron $= \pi h(R^2 - r^2)$

$\pi(36^2 - 27^2) \times 100$

$= 1780.38 cm^3$

Volume of iron = 1780.38cm^3

Mass of 1cm^3 of iron = 7.8gm

Mass of 1780.38cm^3 of iron = 1780.38 × 7.8

= 1388696.4gm

= 1388.7kg

$\therefore$ Mass of roller (m) = 1388.7kg

Q14. The sum of the radius of the base and the height of a solid. cylinder is 37 metres. If the total surface area of the cylinder be 1628 sq metres, find its volume. **5 Marks**

Ans:

Let r and h be the radius and height of the solid cylinder respectively.

Given r + h = 37m

now,

total surface area of the cylimder $= 2\pi r(r + h)$

$\Rightarrow 1628 = 2 \times \frac{22}{7} \times r \times 37$

$\Rightarrow 1628 = \frac{148}{7} \times r$

$\Rightarrow r = \frac{11396}{1628}$

$\Rightarrow r = 7m$

$\Rightarrow r + h = 37$

$\Rightarrow 7 + h = 37$

$\Rightarrow h = 30m$

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 30$

$= 22 \times 7 \times 30$

$= 4620m^3$

Q15. What length of a solid cylinder 2cm in diameter must be taken to recast into a hollow cylinder of length 16cm, external diameter 20 cm and thickness 2.5mm? **5 Marks**

Ans:

We are given a solid cylinder of, diameter = 2cm

We have to recast it into a hollow cylinder of length = 16cm

External Diameter = 20cm and thickness = 2.5mm=0.25cm

We have to find the height of the solid cylinder that can be used to get a hollow cylinder of the desired dimensions.

Volume of a solid cylinder $= \pi r^2 h$

So,

The volume of the given solid cylinder $= \pi(1)^2 h \ldots (a)$

Here, height h has to be found.

Volume of a hollow cylinder $= \pi h(R^2 - r^2)$

Where R is the external radius and r is the internal radius.

External radius is given. Thickness of the hollow cylinder is also given. So, we can find the internal radius of the hollow cylinder.

$\Rightarrow$ Thickness = R−r

$\Rightarrow$ 0.25 = 10 - r

$\Rightarrow$ r = 9.75cm

So, the volume of the hollow cylinder $= \pi \times 16 \times (100 - 95.0.625) \ldots (b)$

From (a) and (b) we get,

$\pi(1)^2 h = \pi \times 16 \times (100 - 95.0625)$

$\pi h = \pi \times 16 \times (100 - 95.0625)$

$h = 16 \times (4.9375)$

$h = 79cm$

Hence, the required height of the solid cylinder is h = 79cm