ATUL CLASSES

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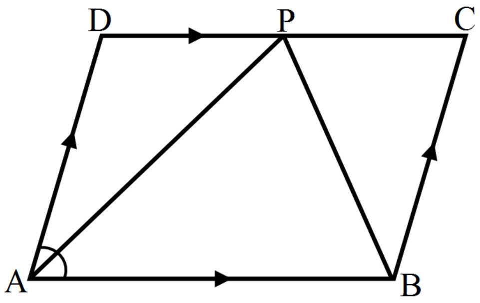
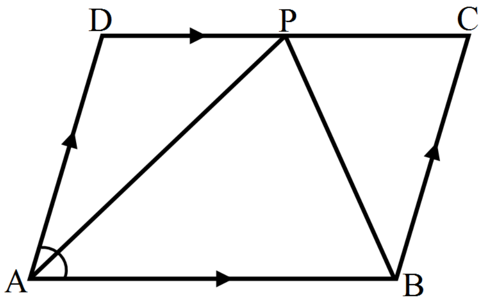
Worksheet Name: Atul Classes **Standard: 9th** **Subject: Mathematics**

Q1. In the adjoining figure, ABCD is a parallelogram in which $\angle A = 60^\circ$. If the bisectors of $\angle A$ and $\angle B$ meet DC at P, prove that

1. $\angle APB = 90^\circ$,
2. AD = DP and PB = PC = BC,
3. DC = 2AD.



Ans:

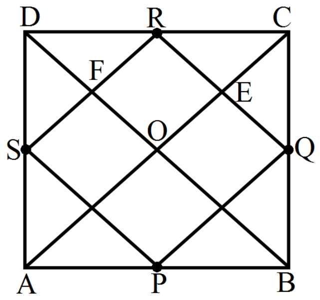


ABCD is a parallelogram in which DA = 60° and bisectore of A and B meets DC at P.
1. In a parallelogram, opposite angles are equal.
So, $\angle C = \angle A = 60^\circ$
In a parallelogram the sum of all the four angles is 360°
$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$
Now, $\angle B + \angle D = 360^\circ - (\angle A + \angle C)$
$= 360^\circ - (60^\circ + 60^\circ) = 240^\circ$
$\therefore 2\angle B = 240^\circ\ [\because \angle B = \angle D]$
So, $\angle B = \angle D = \frac{240^\circ}{2} = 120^\circ$
Since AB || DP and AP is a transversal
SO, $\angle APD = \angle PAB = \frac{60^\circ}{2} = 30^\circ \ldots (1)$ [$\because$ alternate angles]
Also, AB || PC and BP is a transversal.
So, $\angle ABP = \angle CPB$
But, $\angle ABP = \frac{\angle B}{2} = \frac{120^\circ}{2} = 60^\circ$
$\therefore \angle CPB = 60^\circ \ldots (2)$
Now, $\angle APD + \angle APB + \angle CPB = 180^\circ$ [As DPC is a straight line]
$30^\circ + \angle APB + 60^\circ = 180^\circ$
$\Rightarrow \angle APB = 180^\circ - 30^\circ - 60^\circ = 90^\circ$
2. Since $\angle APD = 30^\circ$ [from (1)]
and $\angle DAP = \frac{60^\circ}{2} = 30^\circ$
So, $\angle APD = \angle DAP$
Now in $\triangle APD$,
$\angle APD = \angle DAP \ldots (3)$
$\therefore DP = AD$ [isosceles triangle, sides are equal]
As $\angle CPB = 60^\circ$ [from (2)]
and $\angle C = 60^\circ$
So, $\angle PBC = 180^\circ - 60^\circ - 60^\circ = 60^\circ$
Since all angles in the $\triangle PCB$ are equal,
it is an equilateral triangle.
$\therefore PB = PC = BC \ldots (4)$
3. $\angle DPA = \angle PAD$, [from (3)]
$\therefore DP = AD$ [isoscele striangle, sides are equal]

$= \mathrm{BC}$ [opposite sides are equal]
$= \mathrm{PC}$ [from(4)]
$= \frac{1}{2}\mathrm{DC}$ [$\because \mathrm{DP} = \mathrm{PC} \Rightarrow \mathrm{P}$ is the midpoint of DC]
$\therefore \mathrm{DC} = 2\mathrm{AD}$.

Q2. Show that the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a square is a square.

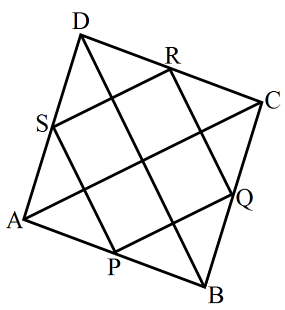
Ans:



Let ABCD be a square and P, Q, R and S be the midpoints of AB, BC, CD and DA,respectively.
Join the diagonals AC and BD. Let BD cut SR at F and AC cut RQ at E. Let O be the intersection point of AC and BD.
In $\triangle\mathrm{ABC}$, we have:
$\therefore \mathrm{PQ} \,||\, \mathrm{AC}$ and $\mathrm{PQ} = \frac{1}{2}\mathrm{AC}$ [By midpoint theorem]
Again, in $\triangle\mathrm{DAC}$,, the points S and R are the midpoints of AD and DC, respectively.
$\therefore \mathrm{SR} \,||\, \mathrm{AC}$ and $\mathrm{SR} = \frac{1}{2}\mathrm{AC}$ [By midpoint theorem]
Now, $\mathrm{PQ} \,||\, \mathrm{AC}$ and $\mathrm{SR} \,||\, \mathrm{AC} \Rightarrow \mathrm{PQ} \,||\, \mathrm{SR}$
Also, $\mathrm{PQ} = \mathrm{SR}$ $\left[\text{Each equal to } \frac{1}{2}\mathrm{AC}\right]$ ...(i)
So, PQRS is a parallelogram.
Now, in $\triangle\mathrm{SAP}$ and $\triangle\mathrm{QBP}$, we have:
$\mathrm{AS} = \mathrm{BQ}$
$\angle\mathrm{A} = \angle\mathrm{B} = 90^\circ$
$\mathrm{AP} = \mathrm{BP}$
i.e., $\triangle\mathrm{SAP} \cong \triangle\mathrm{QBP}$
$\therefore \mathrm{PS} = \mathrm{PQ} \ldots \text{(ii)}$
Similarly, $\triangle\mathrm{SDR} \cong \triangle\mathrm{RCQ}$
$\therefore \mathrm{SR} = \mathrm{RQ} \ldots \text{(iii)}$
From (i), (ii) and (iii), we have:
$\mathrm{PQ} = \mathrm{PS} = \mathrm{SR} = \mathrm{RQ}$ ...(iv)
We know that the diagonals of a square bisect each other at right angles.
$\therefore \angle\mathrm{EOF} = 90^\circ$
Now, $\mathrm{RQ} \,||\, \mathrm{DB}$
$\Rightarrow \mathrm{RE} \,||\, \mathrm{FO}$
Also, $\mathrm{SR} \,||\, \mathrm{AC}$
$\Rightarrow \mathrm{FR} \,||\, \mathrm{OE}$
$\therefore$ OERF is a parallelogram.
So, $\angle\mathrm{FRE} = \angle\mathrm{EOF} = 90^\circ$ (Opposite angles are equal)
Thus, PQRS is a parallelogram with $\angle\mathrm{R} = 90^\circ$ and $\mathrm{PQ} = \mathrm{PS} = \mathrm{SR} = \mathrm{RQ}$.
$\therefore$ PQRS is a square.

Q3. The diagonals of a quadrilateral ABCD are perpendicular to each other. Prove that the quadrilateral formed by joining the midpoints of its sides is a rectangle.

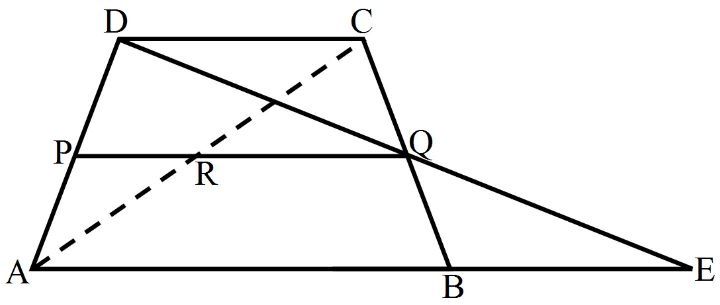
Ans:



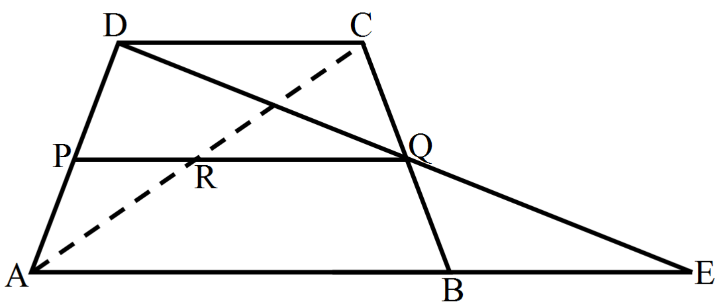
Given: In quadrilateral ABCD, $\mathrm{AC} \perp \mathrm{BD}$, P, Q, R and S are the mid-points of AB, BC, CD and AD, respectively.
To prove: PQRS is a rectangle.
Proof:
In $\triangle\mathrm{ABC}$, P and Q are mid-points of AB and BC, respectively.
$\therefore \mathrm{PQ} \,||\, \mathrm{AC}$ and $\mathrm{PQ} = \frac{1}{2}\mathrm{AC}$ (Mid-point theorem) ...(1)
Similarly, in $\triangle\mathrm{ACD}$,

So, R and S are mid-points of sides CD and AD, respectively.
$\therefore \mathrm{SR} \| \mathrm{AC}$ and $\mathrm{SR} = \frac{1}{2}\mathrm{AC}$ (Mid-point theorem) ...(2)
From (1) and (2), we get
$\mathrm{PQ} \| \mathrm{SR}$ and $\mathrm{PQ} \| \mathrm{SR}$
But this is a pair of opposite sides of the quadrilateral PQRS,
So, PQRS is parallelogram.
Now, in $\triangle\mathrm{BCD}$, Q and R are mid-points of BC and CD, respectively.
$\therefore \mathrm{QR} \| \mathrm{BD}$ and $\mathrm{QR} = \frac{1}{2}\mathrm{BD}$ (Mid-point theorem)...(3)
From (2) and (3), we get
$\mathrm{SR} \| \mathrm{AC}$ and $\mathrm{QR} \| \mathrm{BD}$
But, $\mathrm{AC} \perp \mathrm{BD}$ (Given)
$\therefore \mathrm{RS} \perp \mathrm{QR}$
Hence, PQRS is a rectangle.

Q4. In the adjoining figure, ABCD is a trapezium in which AB || DC and P, Q are the midpoints of AD and BC respectively. DQ and AB when produced meet at E. Also, AC and PQintersect at R. Prove that:
1. DQ = QE,
2. PR || AB,
3. AR = RC.



Ans:
Given: ABCD is trapezium in which AB || DC
P and Q are the mid-points of AD and BC. DQ is joined and produced and Ab is also produced and so that they meet at E.
Ac cuts PQ at R.

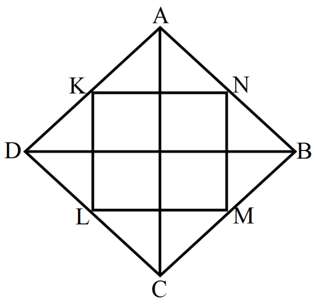


To prove:
DQ = QE
PR || AB
AR = RC
Proof:
1. Consider the triangles $\triangle\mathrm{QCD}$ and $\triangle\mathrm{QBE}$
$\angle\mathrm{DQC} = \angle\mathrm{BQE}$ [verically opposite angles]
$\mathrm{CQ} = \mathrm{BQ}$ [$\because$ Q is the midpoint of BC]
$\angle\mathrm{QDC} = \angle\mathrm{QEB}$ [AE || DC, is a transversal, and thus alternate angles are equal]
Thus, by Angles-Side-Angle criterion of congruence, we have
$\triangle\mathrm{QCD} \cong \triangle\mathrm{QEB}$ [by ASA]
The corresponding parts of the congruent triangles are equal.
Thus, DQ = QE [by C.P.C.T.]
2. Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.
Thus by the midpoint Theorem, PQ || AE.
AB is a part of AE and hence, we have PQ || AB
Since the intercepts made by the lines AB, PQ and DC
on AD
Since PQ || AB || DC
SO, PR which is part of PQ is also parallel to AB
PR || AB || DC

3. Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal. The three lines PR, AB and DC are cut by AC and AD. So, by intercept Theorem, AR = RC

Q5. The diagonals of a quadrilateral ABCD are equal. Prove that the quadrilateral formed by joining the midpoints of its sides is a rhombus.

Ans:



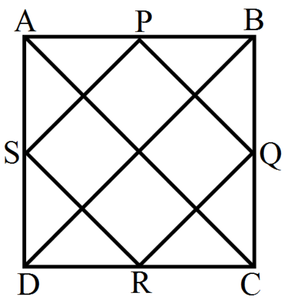
Given: In quadrilateral ABCD, BD = AC and K, L, M and N are the mid-points of AD, CD, BC and AB, respectively.
To prove: KLMN is a rhombus.
Proof:
In $\triangle \mathrm{ADC}$,
Since, K and L are the mid-points of sides AD and CD, respectively.
So, $\mathrm{KL} \,||\, \mathrm{AC}$ and $\mathrm{KL} = \frac{1}{2}\mathrm{AC}\ldots(1)$
Similarly, in $\triangle \mathrm{ABC}$,
Since, M and N are the mid-points of sides BC and AB, respectively.
So, $\mathrm{NM} \,||\, \mathrm{AC}$ and $\mathrm{NM} = \frac{1}{2}\mathrm{AC}\ldots(2)$
From (1) and (2), we get
$\mathrm{KL} = \mathrm{NM}$ and $\mathrm{KL} \,||\, \mathrm{NM}$
But this a pair of opposite sides of the quadrilateral KLMN.
So, KLMN is a parallelogram.
Now, in $\triangle \mathrm{ABD}$,
Since, K and N are the mid-points of sides AD and AB, respectively.
So, $\mathrm{KN} \,||\, \mathrm{BD}$ and $\mathrm{KN} = \frac{1}{2}\mathrm{BD}\ldots(3)$
But $\mathrm{BD} = \mathrm{AC}$ (Given)
$\Rightarrow \frac{1}{2}\mathrm{BD} = \frac{1}{2}\mathrm{AC}$
$\Rightarrow \mathrm{KN} = \mathrm{NM}$ [From (2) and (3)]

But these are a pair of adjacent sides of the parallelogram KLMN.

Hence, KLMN is a rhombus.

Q6. The midpoints of the sides AB, BC, CD and DA of a quadrilateral ABCD are joined to form a quadrilateral. If AC = BD and $\mathrm{AC} \perp \mathrm{BD}$ then prove that the quadrilateral formed is a square.

Ans:



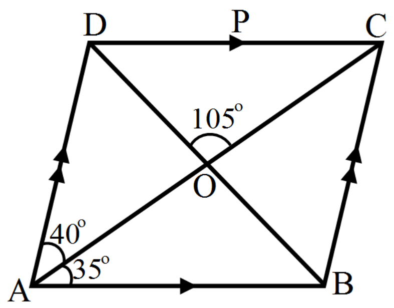
Given: In quadrilateral ABCD, AC = BD and $\mathrm{AC} \perp \mathrm{BD}$. P, Q, R and S are the mid-points of AB, BC, CD and AD, respectively.
To prove: PQRS is a square.
Construction: Join AC and BD.
Proof:
In $\triangle \mathrm{ABC}$,
$\because$ P and Q are mid-points of AB and BC, respectively.
$\therefore \mathrm{PQ} \,||\, \mathrm{AC}$ and $\mathrm{PQ} = \frac{1}{2}\mathrm{AC}$ (Mid-point theorem) ...(1)
Similarly, in $\triangle \mathrm{ACD}$,
$\because$ R and S are mid-points of sides CD and AD, respectively.
$\therefore \mathrm{SR} \,||\, \mathrm{AC}$ and $\mathrm{SR} = \frac{1}{2}\mathrm{AC}$ (Mid-point theorem) ...(2)

From (1) and (2), we get

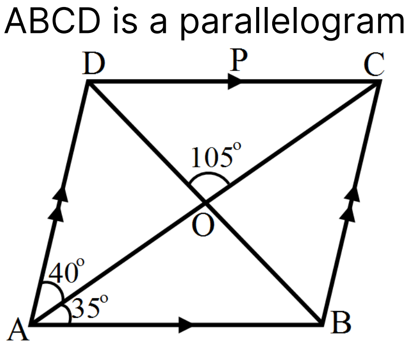
$PQ \parallel SR$ and $PQ = SR$
But this a pair of opposite sides of the quadrilateral PQRS.
So, PQRS is parallelogram.
Now, in $\triangle BCD$,
$\because$ Q and R are mid-points of sides BC and CD, respectively.
$\therefore QR \parallel BD$ and $QR = \frac{1}{2}BD$ (Mid-point theorem) ...(3)
From (2) and (3), we get
$RS \parallel AC$ and $QR \parallel BD$
But, $AC \perp BD$ (Given)
$\therefore RS \perp QR$
But this a pair of adjacent sides of the parallelogram PQRS.
So, PQRS is a rectangle.
Again, AC = BD (Given)
$\Rightarrow RS = QR$ [From (2) and (3)]
But this a pair of adjacent sides of the rectangle PQRS.
Hence, PQRS is a square.

Q7. In the adjoining figure, ABCD is a parallelogram in which $\angle BAO = 35^\circ, \angle DAO = 40^\circ$ and $\angle COD = 150^\circ$. Calculate

1. $\angle ABO$,
2. $\angle ODC$,
3. $\angle ACB$,
4. $\angle CBD$.



Ans:
ABCD is a parallelogram

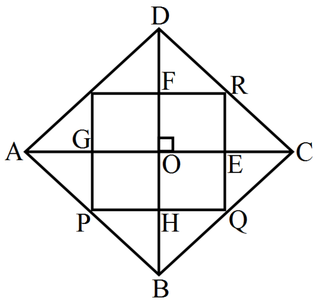


1. $\angle AOB = \angle COB = 105^\circ$ [Vertical opposite angels]
Now in $\triangle AOB$, we have
$\angle OAB + \angle AOB + \angle ABO = 180^\circ$
$\Rightarrow 35^\circ + 105^\circ + \angle ABO = 180^\circ$
$\Rightarrow 140^\circ + \angle ABO = 180^\circ$
$\Rightarrow \angle ABO = 180^\circ - 140^\circ = 40^\circ$.
2. Since AB || DC and BD is a transversal
So, $\angle ABD = \angle CDB$ [alternate angles]
$\Rightarrow \angle CDO = \angle CDB = \angle ABD = \angle ABO = 40^\circ$
$\therefore \angle ODC = 40^\circ$
3. As AB || CD and AC is a transversal
So, $\angle ACB = \angle DAC = 40^\circ$ [alternate opposite angles]
4. $\angle CBD = \angle B - \angle ABO$
But, $\angle A + \angle B + \angle C + \angle D = 360^\circ$ [$\because$ ABCD is a parrellogram]
$\Rightarrow 2\angle A + 2\angle B = 360^\circ$
$\Rightarrow 2 \times (40^\circ + 35^\circ) + 2\angle B = 360^\circ$
$\Rightarrow 150^\circ + 2\angle B = 360^\circ$
$\Rightarrow 2\angle B = 360^\circ - 150^\circ = 210^\circ$
$\Rightarrow \angle B = \frac{210^\circ}{2} = 105^\circ$
and $\angle CBD = \angle B - \angle ABO$
$= 105^\circ - 40^\circ = 65^\circ$

$\angle CBD = 65^{\circ}$

Q8. Show that the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rhombus is a rectangle.

Ans:



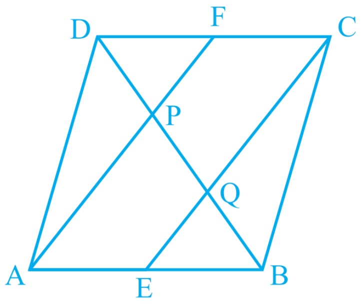
Let ABCD be a rhombus and P, Q, R and S be the midpoints of AB, BC, CD and DA, respectively.
Join the diagonals, AC and BD.
In $\triangle ABC$, we have:
$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ [By midpoint theorem]
Again, in $\triangle DAC$, the points S and R are the midpoints of AD and DC, respectively.
$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ [By midpoint theorem]
Now, $PQ \parallel AC$ and $SR \parallel AC \Rightarrow PQ \parallel SR$
Also, $PQ = SR$ $\left[\text{Each equal to } \frac{1}{2}AC\right]$ ...(i)
So, PQRS is a parallelogram.
We know that the diagonals of a rhombus bisect each other at right angles.
$\therefore \angle EOF = 90^{\circ}$
Now, $RQ \parallel DB$
$\Rightarrow RE \parallel FO$
Also, $SR \parallel AC$
$\Rightarrow FR \parallel OE$
$\therefore$ OERF is a parallelogram.
So, $\angle FRE = \angle EOF = 90^{\circ}$ (Opposite angles are equal)
Thus, PQRS is a parallelogram with $\angle R = 90^{\circ}$.
$\therefore$ PQRS is a rectangle.

Q9. In a $||$ gm ABCD, if $\angle A = (2x + 25)^{\circ}$ and $\angle B = (3x - 5)^{\circ}$, find the value of x and the measure of each angle of the parallelogram.

Ans:

In a parallelogarm, the opposite angle are equal.
So, in the parallelogram ABCD,
$\angle A = \angle C$
and $\angle B = \angle D$
Since $\angle A = (2x + 25)^{\circ}$
$\therefore \angle C = (2x + 25)^{\circ}$
and $\angle B = (3x - 5)^{\circ}$
$\therefore \angle D = (3x - 5)^{\circ}$
In a parallelogram, the sum of all the four angles is 360°
$\therefore \angle A + \angle B + \angle C + \angle D = 360^{\circ}$
$\Rightarrow (2x + 25) + (3x + 5) + (2x + 25) + (3x - 5) = 360^{\circ}$
$\Rightarrow 10x + 40 = 360^{\circ}$
$\Rightarrow 10x = 360^{\circ} - 40^{\circ} = 320^{\circ}$
$\Rightarrow x = \frac{320}{10} = 32^{\circ}$
$\therefore \angle A = (2x + 25) = (2 \times 32 + 25) = 89^{\circ}$
$\angle B = (3x - 5) = (3 \times 32 - 5) = 91^{\circ}$
$\angle C = (2x + 25) = (2 \times 32 + 25) = 89^{\circ}$
$\angle D = (3x - 5) = (3 \times 32 - 5) = 91^{\circ}$
$\therefore \angle A = \angle C = 89^{\circ}$ and $\angle B = \angle D = 91^{\circ}$

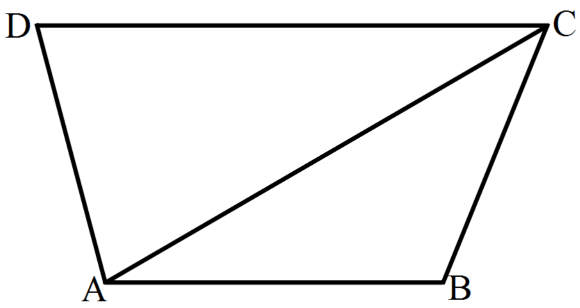
Q10. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. Show that the line segments AF and EC trisect the diagonal BD.



Ans:

Given,
ABCD is a parallelogram. E and F are the mid-points of sides AB and CD respectively.
To show,
AF and EC trisect the diagonal BD.
Proof,
ABCD is a parallelogram
Therefore, AB || CD
also, AE || FC
Now,
AB = CD (Opposite sides of parallelogram ABCD)
$\Rightarrow \frac{1}{2}\mathrm{AB} = \frac{1}{2}\mathrm{CD}$
⇒ AE = FC (E and F are midpoints of side AB and CD)
AECF is a parallelogram (AE and CF are parallel and equal to each other)
AF || EC (Opposite sides of a parallelogram)
Now,
In $\Delta\mathrm{DQC}$,
F is midpoint of side DC and FP || CQ (as AF || EC).
P is the mid-point of DQ (Converse of mid-point theorem)
⇒ DP = PQ ... (i)
Similarly,
In APB,
E is midpoint of side AB and EQ || AP (as AF || EC).
Q is the mid-point of PB (Converse of mid-point theorem)
⇒ PQ = QB ... (ii)
From equations (i) and (i),
DP = PQ = BQ
Hence, the line segments AF and EC trisect the diagonal BD.

Q11. In the adjoining figure, ABCD is a quadrilateral and AC is one of its diagonals. Prove that:
(i) AB + BC + CD + DA > 2AC
(ii) AB + BC + CD > DA
(iii) AB + BC + CD + DA > AC + BD



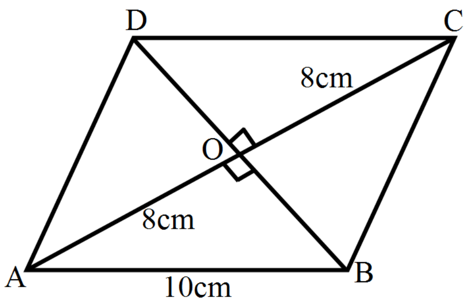
Ans:

Given: ABCD is a quadrilateral and AC is one of its diagonal.
1. We know that the sum of any two sides of a triangle is greater than the third side.
In $\triangle\mathrm{ABC}$, AB + BC > AC ...(1)
In $\triangle\mathrm{ACD}$, CD + DA > AC ...(2)
Adding inequalities (1) and (2), we get:
AB + BC + CD + DA > 2AC
2. In $\triangle\mathrm{ABC}$, we have:
AB + BC > AC ...(1)
We also know that the length of each side of a triangle is greater than the positive difference of the length of the other two sides.
In $\triangle\mathrm{ACD}$, we have:
AC > |DA − CD| ...(2)
From (1) and (2), we have:

AB + BC > |DA – CD|
⇒ AB + BC + CD > DA
3. In $\triangle ABC$, $AB + BC > AC$
In $\triangle ACD$, $CD + DA > AC$
In $\triangle BCD$, $BC + CD > BD$
In $\triangle ABD$, $DA + AB > BD$
Adding these inequalities, we get:
2(AB + BC + CD + DA) > 2(AC + BD)
⇒ (AB + BC + CD + DA) > (AC + BD)

Q12. Each side of a rhombus is 10cm long and one of its diagonals measures 16cm. Find the length of the other diagonal and hence find the area of the rhombus.

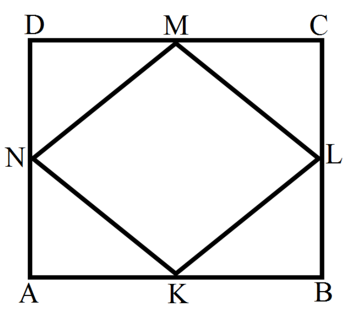
Ans:
Since diagonals of a rhombus bisect each other at right angles.
So, $AO = OC = \frac{1}{2}AC = \frac{1}{2} \times 16 = 8cm.$
$\therefore$ In right $\triangle AOB$,
$AB^2 = AO^2 + OB^2$
$\Rightarrow 10^2 = 8^2 + OB^2$
$\Rightarrow OB^2 = 100 - 64 = 36$
$\Rightarrow OB = \sqrt{36} = 6cm.$
$\therefore$ Length of the other diagonal $BD = 2 \times OB$
$= 2 \times 6 = 12cm.$



Area of $\triangle ABC = \frac{1}{2} \times AC \times OB$
$= \frac{1}{2} \times 16 \times 6 = 48cm^2.$
Area of $\angle ACD = \frac{1}{2} \times AC \times OD$
$= \frac{1}{2} \times 16 \times 6 = 48cm^2.$
$\therefore$ Area of rhombus ABCD = $(\text{Area of } \triangle ABC + \text{Area of } \triangle ACD)$
$= (48 + 48)cm^2 = 96cm^2.$

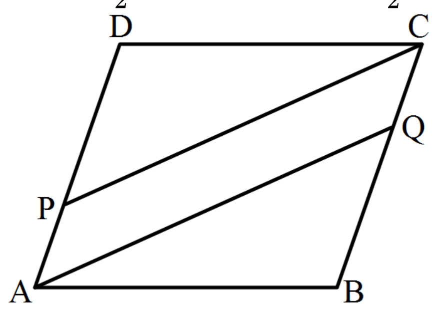
Q13. K, L, M and N are points on the sides AB, BC, CD and DA respectively of a square ABCD such that AK = BL = CM = DN. Prove that KLMN is a square.

Ans:



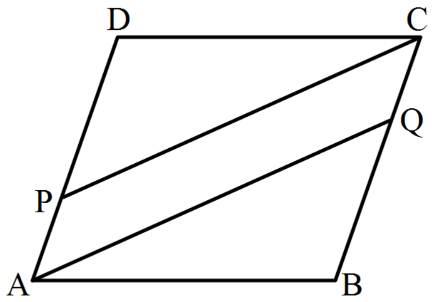
$AK = BL = CM = DN$ (given)
$\Rightarrow BK = CL = DM = AN\ldots(1)$ (since ABCD is a square)
In $\triangle AKN$ and $\triangle BLK$,
$AK = BL$ (given)
$\angle A = \angle B$ (Each 90°)
$AN = BK$ [From (i)]
$\therefore \triangle AKN \cong \triangle BLK$ (by SAS congruence criterion)
$\Rightarrow \angle AKN = \angle BLK$ and $\angle ANK = \angle BKL$ (C.P.C.T.)
But, $\angle AKN + \angle ANK = 90^\circ$ and $\angle BLK + \angle BKL = 90^\circ$
$\Rightarrow \angle AKN + \angle ANK + \angle BLK + \angle BKL = 90^\circ + 90^\circ$
$\Rightarrow 2\angle AKN + 2\angle BKL = 180^\circ$
$\Rightarrow \angle AKN + \angle BKL = 90^\circ$
$\Rightarrow \angle NKL = 90^\circ$
Similarly, we have
$\angle KLM = \angle LMN = \angle MNK = 90^\circ$
Hence, KLMN is a square.

Q14. In the adjoining figure, ABCD is a parallelogram. If P and Q are points on AD and BCrespectively such that $\mathrm{AP} = \frac{1}{2}\mathrm{AD}$ and $\mathrm{CQ} = \frac{1}{2}\mathrm{BC}$, prove that AQCP is a parallelogram.



Ans:

Given: A parallelogram ABCD in which $\mathrm{AP} = \frac{1}{3}\mathrm{AD}$ and $\mathrm{CQ} = \frac{1}{3}\mathrm{BC}$



To Prove: PAQC is a parallelogram.
Proof: In $\triangle\mathrm{ABQ}$ and $\triangle\mathrm{CDP}$
$\mathrm{AB} = \mathrm{CD}$ [∵ Opposite angles of parallelogram]
$\angle\mathrm{B} = \angle\mathrm{D}$
and $\mathrm{DP} = \mathrm{AD} - \mathrm{PA} = \frac{2}{3}\mathrm{AD}$
and, $\mathrm{BQ} = \mathrm{BC} - \mathrm{CQ} = \mathrm{BC} - \frac{1}{3}\mathrm{BC}$
$= \frac{2}{3}\mathrm{BC} = \frac{2}{3}\mathrm{AD}$ [∵ AD = BC]
$\therefore \mathrm{BQ} = \mathrm{DP}$
Thus, by Side-Angle-Side criterion of congruence, we have,
So, $\triangle\mathrm{ABQ} \cong \triangle\mathrm{CDP}$ [By SAS]
The corresponding parts of the congruent triangles are equal.
$\mathrm{AQ} = \mathrm{CP}$ [By C.P.C.T.]
and $\mathrm{PA} = \frac{1}{3}\mathrm{AD}$
and $\mathrm{CQ} = \frac{1}{3}\mathrm{BC} = \frac{1}{3}\mathrm{AD}$
$\mathrm{PA} = \mathrm{CQ}$ $[\because \mathrm{AD} = \mathrm{BC}]$
Also, by C.P.C.T., $\angle\mathrm{QAB} = \angle\mathrm{PCD}\ldots(1)$
Therefore,
$\angle\mathrm{QAP} = \angle\mathrm{A} - \angle\mathrm{QAB}$
$= \angle\mathrm{C} - \angle\mathrm{PCD}$ [Since $\angle\mathrm{A} = \angle\mathrm{C}$ and from (1)]
$= \angle\mathrm{PCQ}$ [alternate interior angles are equal]
Therefore, AQ and CP are two parallel lines.
So, PAQC is a parallelogram.

Q15. Show that the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rectangle is a rhombus.

Ans:

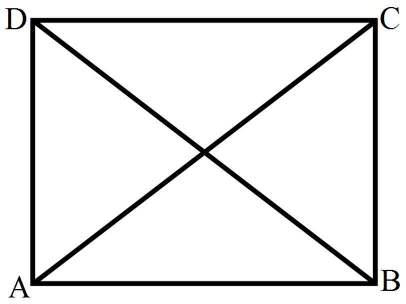
Let ABCD be the rectangle and P, Q, R and S be the midpoints of AB, BC, CD and DA, respectively.
Join AC, a diagonal of the rectangle.
In $\triangle\mathrm{ABC}$, we have:
∴ PQ || AC and $\mathrm{PQ} = \frac{1}{2}\mathrm{AC}$ [By midpoint theorem]
Again, in $\triangle\mathrm{DAC}$, the points S and R are the mid points of AD and DC, respectively.
∴ SR || AC and $\mathrm{SR} = \frac{1}{2}\mathrm{AC}$ [By midpoint theorem]
Now, PQ || AC and SR || AC
⇒ PQ || SR
Also, PQ = SR [Each equal to $\frac{1}{2}$ AC] ...(i)
So, PQRS is a parallelogram.
Now, in $\triangle\mathrm{SAP}$ and $\triangle\mathrm{QBP}$, we have:
AS = BQ
$\angle\mathrm{A} = \angle\mathrm{B} = 90^\circ$
AP = BP
i.e., $\triangle\mathrm{SAP} \cong \triangle\mathrm{QBP}$
∴ PS = PQ ...(ii)

Similarly, $\triangle SDR \cong \triangle QCR$
$\therefore$ SR = RQ ...(iii)
From (i), (ii) and (iii), we have:
PQ = PQ = SR = RQ
Hence, PQRS is a rhombus.

Q16. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

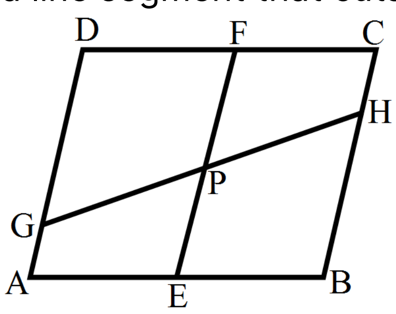
1. ABCD is a square
2. diagonal BD bisects $\angle B$ as well as $\angle D$.

Ans:

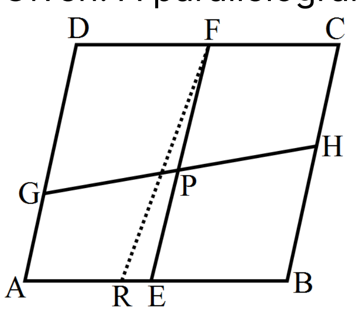


1. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.
$\Rightarrow \angle BAC = \angle DAC \ldots (1)$
And $\angle BCA = \angle DCA \ldots (2)$
Since every rectangle is a parallelogram, therefore
AB || DC and AC is the transversal.
$\Rightarrow \angle BAC = \angle DCA$ (alternate angles)
$\Rightarrow \angle DAC = \angle DCA$ [from]
Thus in $\triangle ADC$,
$AD = CD$ (opposite sides of equal angles are equal)
But, $AD = BC$ and $CD = AB$ (ABCD is a rectangle)
$\Rightarrow AB = BC = CD = AD$
Hence, ABCD is a square.
2. In $\triangle BAD$ and $\triangle BCD$,
$AB = CD$
$AD = BC$
$BD = BD$
$\therefore \triangle BAD \cong \triangle BCD$ (by SSS congruence criterion)
$\Rightarrow \angle ABD = \angle CBD$ and $\angle ADB = \angle CDB$ [CP.C.T.]
Hence, diagonal BD bisects $\angle B$ as well as $\angle D$.

Q17. In the adjoining figure, ABCD is a || gm in which E and F are the midpoints of AB and CDrespectively. If GH is a line segment that cuts AD, EF and BC at G, P and H respectively, prove that GP = PH.



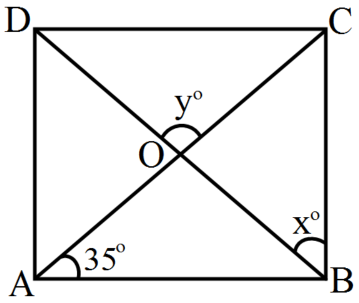
Ans:
Given: A parallelogram ABCD in which E and F are the mid points of AB and CD. A line segment GH cuts EF at P.



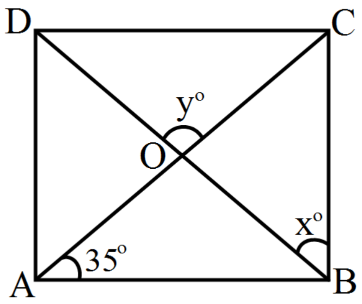
To prove: GP = PH
Proof: AD, EF and BC are three line segments and DC and Ab are two transversal.
The intercepts made by the line on transversal AB and CD are equal because,
AE = EB
and DF = FC
We need to prove that FE is parallel to AD.
Let us prove by the method of contradiction.
Let us assume that FE is not parallel to AD.

Now, draw FR parallel to AD.
Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.
Thus, by Intercept Theorem, AR = RB because
DF = FC
But AE = EB [Given]
There can not be two mid points R and E of AB. Hence our assumption is wrong.
SO, AD || EF || BC
Now, again by Intercept Theorem, we have
GP = PH
because GH is transversal and intecept made by AD, EF and BC on are as DF = FC.

Q18. In each of the figures given below, ABD is a rectangle. Find the values of x and y in each case.

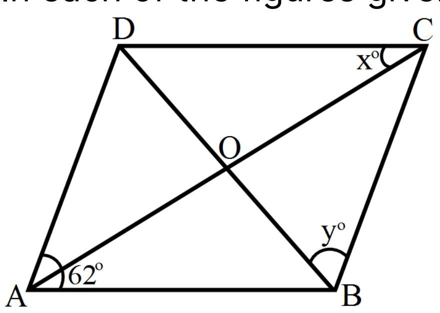


Ans:

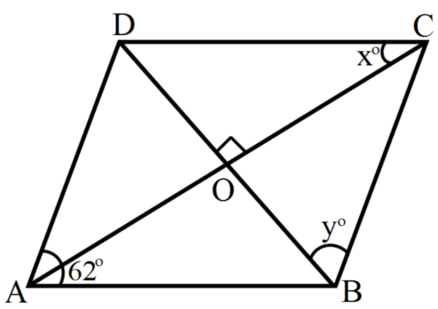


We know that diagonals of a rectangle are equal and bisect each other.
So, in $\triangle \mathrm{AOB}$
$\mathrm{AO} = \mathrm{OB}$
$\Rightarrow \angle \mathrm{OAB} = \angle \mathrm{OBA}$ [base angles are equal]
i.e. $\angle \mathrm{OBA} = 35^\circ$ $[\because \angle \mathrm{OAB} = 35^\circ, \text{given}]$
$\angle \mathrm{AOB} = 180^\circ - 35^\circ - 35^\circ = 110^\circ$
and, $\angle \mathrm{DOC} = \mathrm{y}^\circ = \angle \mathrm{AOB} = 110^\circ$ [Vertically opp. angle]
Consider the right triangle, $\triangle \mathrm{ABC}$, right angle at B.
So, $\angle \mathrm{ABC} = 90^\circ$ [$\because$ ABCD is a rectangle]
Now, consider the $\triangle \mathrm{OBC}$
So, $\angle \mathrm{OBC} = \mathrm{x}^\circ = \angle \mathrm{ABC} - \angle \mathrm{OBA}$
$= 90^\circ - 35^\circ$
$= 55^\circ$
$\therefore \mathrm{x} = 55^\circ$ and $\therefore \mathrm{y} = 110^\circ$.

Q19. In each of the figures given below, ABCD is a rhombus. Find the value of x and y in each case.



Ans:
Since ABCD is a rhombus

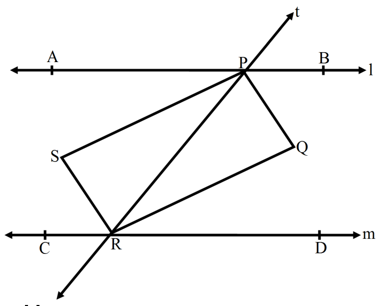


So, $\angle \mathrm{A} = \angle \mathrm{C}$, i.e., $\angle \mathrm{C} = 62^\circ$
Now in $\triangle \mathrm{BCD}, \mathrm{BC} = \mathrm{DC}$
$\Rightarrow \angle \mathrm{CDB} = \angle \mathrm{DBC} = \mathrm{y}^\circ$
As, $\angle \mathrm{BDC} + \angle \mathrm{DBC} + \angle \mathrm{BCD} = 180^\circ$
$\Rightarrow \mathrm{y} + \mathrm{y} + 62^\circ = 180^\circ$

$\Rightarrow 2y = 180^\circ - 62^\circ = 118^\circ$
$\Rightarrow y = \frac{118}{2} = 59^\circ$
As diagonals of a rhombus are perpendicular to each other,
$\triangle$OCD is a right triangle and $\angle DOC = 90^\circ, \angle ODC = y = 59^\circ$
$\Rightarrow \angle DCO = 90^\circ - \angle ODC$
$= 90^\circ - 59^\circ = 31^\circ$
$\therefore \angle DCO = x = 31^\circ$
$\therefore x = 31^\circ$ and $y = 59^\circ$

Q20. Two parallel lines l and m are intersected by a transversal t. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Ans:



l || m and t is a transversal.
$\Rightarrow \angle APR = \angle PRD$ (alternate angles)
$\Rightarrow \frac{1}{2}\angle APR = \frac{1}{2}\angle PRD$
$\Rightarrow \angle SPR = \angle PRQ$ (PS and RQ are the bisectors of $\angle APR$ and $\angle PRD$)
Thus, PR intersects PS and RQ at P and R respectively such that $\angle SPR = \angle PRQ$ i.e., alternate angles are equal.
$\Rightarrow PS \parallel RQ$
Similarly, we have $SR \parallel PQ$.
Hence, PQRS is a parallelogram.
Now, $\angle BPR + \angle PRD = 180^\circ$ (interior angles are supplementary)
$\Rightarrow 2\angle QPR + 2\angle QRP = 180^\circ$ (PQ and RQ are the bisectors of $\angle BPR$ and $\angle PRD$)
$\Rightarrow \angle QPR + \angle QRP = 90^\circ$
In $\triangle PQR$, by angle sum property,
$\angle PQR + \angle QPR + \angle QRP = 180^\circ$
$\Rightarrow \angle PQR + 90^\circ = 180^\circ$
$\Rightarrow \angle PQR = 90^\circ$
Since PQRS is a parallelogram,
$\angle PQR = \angle PSR$
$\Rightarrow \angle PSR = 90^\circ$
Now, $\angle SPQ + \angle PQR = 180^\circ$ (adjacent angles in a parallelogram are supplementary)
$\Rightarrow \angle SPQ + 90^\circ = 180^\circ$
$\Rightarrow \angle SPQ = 90^\circ$
$\Rightarrow \angle SRQ = 90^\circ$
Thus, all the interior angles of quadrilateral PQRS are right angles.
Hence, PQRS is a rectangle.

Q21. If an angle of a parallelogram is four-fifths of its adjacent angle, find the angles of the parallelogram.

Ans:

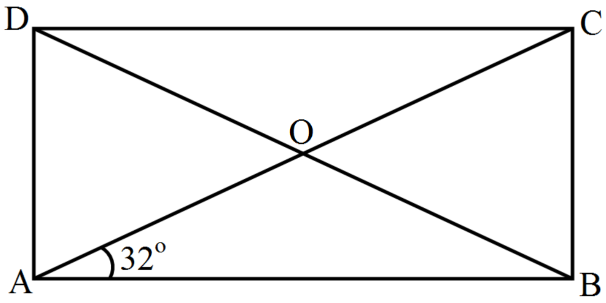
Let ABCD be a parallelogram.
Suppose, $\angle A = x^\circ$
Then, $\angle B$, which is adjacent angle of A is $\frac{4}{5}x^\circ$.
In a parallelogram, the opposite angles are equal
$\Rightarrow \angle A = \angle C = x^\circ$ and $\angle B = \angle D = \frac{4}{5}x^\circ$
The sum of all the four angles of a parallelogram is 360°.
$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$
$\Rightarrow x + \frac{4}{5}x + x + \frac{4}{5}x = 360^\circ$
$\Rightarrow 2x + \frac{8}{5}x = 360^\circ$
$\Rightarrow \frac{18}{5}x = 360^\circ$
$\Rightarrow x = \frac{360 \times 5}{18} = 100^\circ$
$\therefore \angle A = x = 100^\circ$
$\angle B = \frac{4}{5}x = \frac{4}{5} \times 100 = 80^\circ$
$\angle C = x = 100^\circ$

$\angle D = \frac{4}{5}x = \frac{4}{5} \times 100 = 80^\circ$
$\therefore \angle A = \angle C = 100^\circ$ and $\angle B = \angle D = 80^\circ$.

Q22. If ABCD is a rectangle with $\angle BAC = 32^\circ$, find the measure of $\angle DBC$.

Ans:

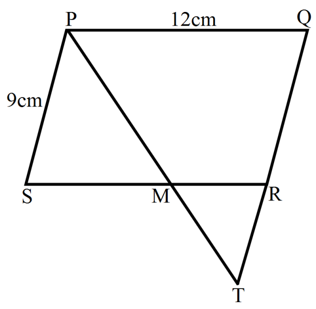
Figure is given as :



Suppose the diagonals AC and BD intersect at O.
Since, diagonals of a rectangle are equal and they bisect each other.
Therefore, in $\triangle OAB$, we have
OA = OB
Angles opposite to equal sides are equal.
Therefore,
$\angle OAB = \angle OBA$
$\angle BAC = \angle DBA$
But, $\angle BAC = 32^\circ$
$\angle DBA = 32^\circ$
Now,
$\angle ABC = 90^\circ$
$\angle DBA + \angle DBC = 90^\circ$
$32^\circ + \angle DBC = 90^\circ$
$\angle DBC = 58^\circ$
Hence, the measure of $\angle DBC$ is 58**°**.

Q23. In a parallelogram PQRS, PQ = 12cm and PS = 9cm. The bisector of $\angle P$ meets SR in M. PM and QR both when produced meet at T. Find the length of RT.

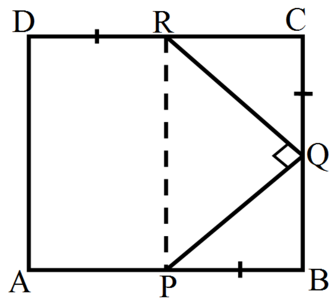
Ans:



PM is the bisector of $\angle P$.
$\Rightarrow \angle QPM = \angle SPM \ldots (i)$
PQRS is a parallelogram.
$\therefore PQ \,||\, SR$ and PM is the transversal.
$\Rightarrow \angle QPM = \angle MS \ldots (ii)$ (alternate angles)
From (i) and (ii),
$\angle SPM = \angle PMS$ ...(iii)
$\Rightarrow MS = PS = 9cm$ (sides opposite to equal angles are equal)
Now, $\angle RMT = \angle PMS$ ...(iv) (vertically opposite angles)
Also, $PS \,||\, QT$ and PT is the transversal.
$\angle RTM = \angle SPM$
$\Rightarrow \angle RTM = \angle RMT$
$\Rightarrow RT = RM$ (sides opposite to equal angles are equal)
$RM = SR - MS = 12 - 9 = 3cm$
$\Rightarrow RT = 3cm$

Q24. In the given figure, ABCD is a square and $\angle PQR = 90^\circ$. If PB = QC = DR, prove that:

1. QB = RC,
2. PQ = QR,
3. $\angle QPR = 45^\circ$



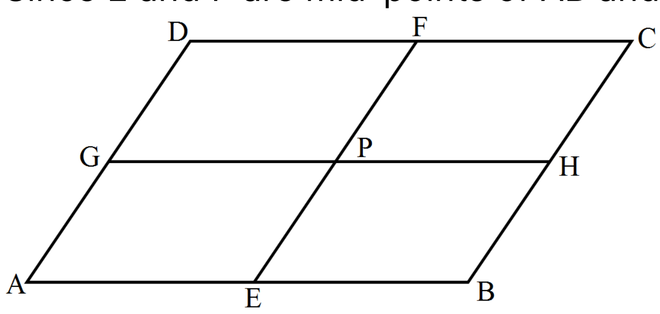
Ans:

Given: ABCD is a square and $\angle \mathrm{PQR} = 90^{\circ}$.
Also, PB = QC = DR
1. We have:
BC = CD (Sides of square)
CQ = DR (Given)
BC = BQ + CQ
⇒ CQ = BC − B
∴ DR = BC − BQ ...(i)
Also, CD = RC+ DR
∴ DR = CD − RC = BC − RC ...(ii)
From (i) and (ii), we have:
BC − BQ = BC − RC
∴ BQ = RC
2. In $\triangle \mathrm{RCQ}$ and $\triangle \mathrm{QBP}$, we have:
$\mathrm{PB} = \mathrm{QC}$ (Given)
$\mathrm{BQ} = \mathrm{RC}$ (Proven above)
$\angle \mathrm{RCQ} = \angle \mathrm{QBP}$ (90° each)
i.e., $\triangle \mathrm{RCQ} \cong \triangle \mathrm{QBP}$ (SAS congruence rule)
∴ $\mathrm{QR} = \mathrm{PQ}$ (By C.P.C.T.)
3. $\triangle \mathrm{RCQ} \cong \triangle \mathrm{QBP}$ and $\mathrm{QR} = \mathrm{PQ}$ (Prove above)
∴ in $\triangle \mathrm{RPQ}, \angle \mathrm{QRP} = \angle \mathrm{QRP} = \frac{1}{2}(180^{\circ} - 90^{\circ}) = \frac{90^{\circ}}{2} = 45^{\circ}$

Q25. ABCD is a parallelogram; E and f are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC at G, P and H respectively. Prove that GP = PH.

Ans:

Since E and F are mid-points of AB and CD respectively



$\mathrm{AE} = \mathrm{BE} = \left(\frac{1}{2}\right)\mathrm{AB}$
And $\mathrm{CF} = \mathrm{DF} = \left(\frac{1}{2}\right)\mathrm{CD}$
But, AB = CD
$\left(\frac{1}{2}\right)\mathrm{AB} = \left(\frac{1}{2}\right)\mathrm{CD}$
⇒ BE = CF
Also, BE || CF [∴ AB || CD]
Therefore, BEFC is a parallelogram
BC || EF and BE = PH ....(i)
Now, BC || EF
⇒ AD || EF [∴ BC || AD as ABCD is a parallelogram]
Therefore, AEFD is a parallelogram.
⇒ AE = GP
But E is the mid-point of AB.
So, AE = BF
Therefore, GP = PH.